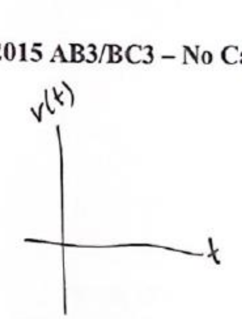
SOLUTIONS

AP Calculus AB – Worksheet 87 | **Particle Motion – Tables and Graphs**

2015 AB3/BC3 – No Calculator



16

150

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	−220	150

continuous smooth graph

3. Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

   (a) Use the data in the table to estimate the value of $v'(16)$.

   (b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

   (c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by velocity → $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

   (d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

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a) $v'(16) \approx \dfrac{v(20)-v(12)}{20-12} = \dfrac{240-200}{20-12}\ \dfrac{\text{meters}/\text{min}}{\text{min}}$

b) $\int_0^{40} |v(t)|\,dt$ represents the total distance, in meters, that Johanna traveled from $t=0$ to $t=40$ minutes.

$$v(40)\cdot 16 + |v(24)\cdot 4| + v(20)\cdot 8 + v(12)\cdot 12$$

$$(150)(16) + (220)(4) + (240)(8) + (200)(12)$$

c) $B'(t) = 3t^2 - 12t$

$B'(5) = 3(5)^2 - 12(5)$

d) Average Velocity $= \dfrac{\text{displacement}}{\text{time}}$

$$= \frac{\int_0^{10} [t^3 - 6t^2 + 300]\,dt}{10}$$

$$= \frac{\left[\frac{1}{4}t^4 - 2t^3 + 300t + C\right]_0^{10}}{10} = \frac{\frac{1}{4}(10)^4 - 2(10)^3 + 300(10)}{10}$$

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters / minute)	0	100	40	−120	−150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

   (a) Find the average acceleration of train A over the interval $2 \le t \le 8$.

   (b) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

   (c) At time $t = 2$, train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.

   (d) A second train, train B, travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

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a) $\text{Avg. Accel} = \dfrac{V_A(8) - V_A(2)}{8-2} = \dfrac{-120-100}{6} = -\dfrac{220}{6} = -\dfrac{110}{3}\ \text{m}/\text{min}^2$

b) $V_A(5) = 40 > -100$

$V_A(8) = -120 < -100$

Since $V_A(t)$ is differentiable, then, by IVT, $V_A(t) = -100$ in $(5, 8)$.

c) $S_A(2) = 300$

$S_A(12) = 300 + \int_2^{12} V_A(t)\,dt$

$= 300 + \frac{1}{2}\left[V_A(2) + V_A(5)\right] \cdot 3 + \frac{1}{2}\left[V_A(5) + V_A(8)\right] \cdot 3 + \frac{1}{2}\left[V_A(8) + V_A(12)\right] \cdot 4$

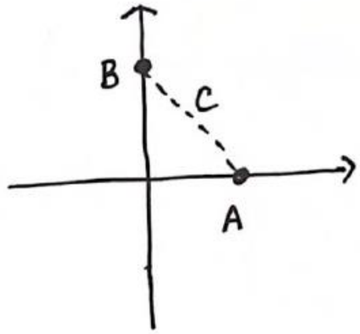
$= 300 + \frac{3}{2}\left[140\right] + \frac{3}{2}\left[-80\right] + 2\left[-270\right]$

$= 300 + 210 - 120 - 540$

$= -150$

d) $V_B(t) = -5t^2 + 60t + 25$

$S_B(2) = 400$



MOMENT $(t=2)$

$A = 300$

$B = 400$

$C = 500$

RATES

$V_A(2) = 100$

$V_B(2) = 125$

$$A^2 + B^2 = C^2$$

$$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2C\frac{dC}{dt}$$

$$300(100) + 400(125) = 500\frac{dC}{dt}$$

$$\frac{300 + 500}{5} = \frac{dC}{dt}$$

$$160 \text{ m/min} = \frac{dC}{dt}$$

The distance between TRAIN A & TRAIN B is increasing at a rate of 160 m/min at t=2 min.

The data in the table below gives selected values for the velocity, in meters/minute, of a particle moving along the x-axis. The velocity v is a differentiable function of time t.

Time t (min)	0	2	5	6	8	12
Velocity $v(t)$ (meters/min)	-3	2	3	5	7	5

a) At $t=5$, is the particle moving to the right or to the left? Justify.

$v(5)=3>0 \rightarrow$ The particle is moving to the right since $v(5)>0$.

b) Is there a time during the time interval $0 \le t \le 12$ at which the particle is at rest? Explain your reasoning.

At rest $\rightarrow v(t)=0$

Since $v(t)$ is differentiable at $v(0)=-3<0$ & $v(2)=2>0$,

the $v(t)=0$ in $(0,2)$.

c) Use the data from the table to approximate the acceleration of the particle at $t=10$. Show the computations that lead to your answer.

$$a(10) \approx \frac{v(12)-v(8)}{12-8} = \frac{5-7}{4} = -\frac{1}{2} \text{ m/min}^2$$

d) Find the average acceleration of the particle over the time interval $8 \le t \le 12$. Explain what this answer means in terms of the particle's velocity.

$$\text{Avg Accel} = \frac{v(12)-v(8)}{12-8} = -\frac{1}{2} \text{ m/min}^2.$$ The velocity is decreasing

e) Use a right Riemann sum with 5 subintervals to find the value of $\int_0^{12} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} v(t)\,dt$ in terms of the particle.

$\int_0^{12} v(t)dt$ represents the displacement of the particle from $t=0$ to $t=12$ min.

$$\int_0^{12} v(t)dt \approx 4\cdot v(12) + 2v(8) + 1\cdot v(6) + 3\cdot v(5) + 2\cdot v(2)$$
$$= 20 + 14 + 5 + 9 + 4$$
$$= 52 \text{ meters}$$

f) Explain why there must be a value, c, in the interval $0 \le t \le 12$ such that $a(c)=0$

Since $a(c) = \dfrac{v(b)-v(a)}{b-a}$ for some value, by MVT,

$$a(c) = \frac{v(12)-v(6)}{12-6} = \frac{5-5}{6} = 0$$